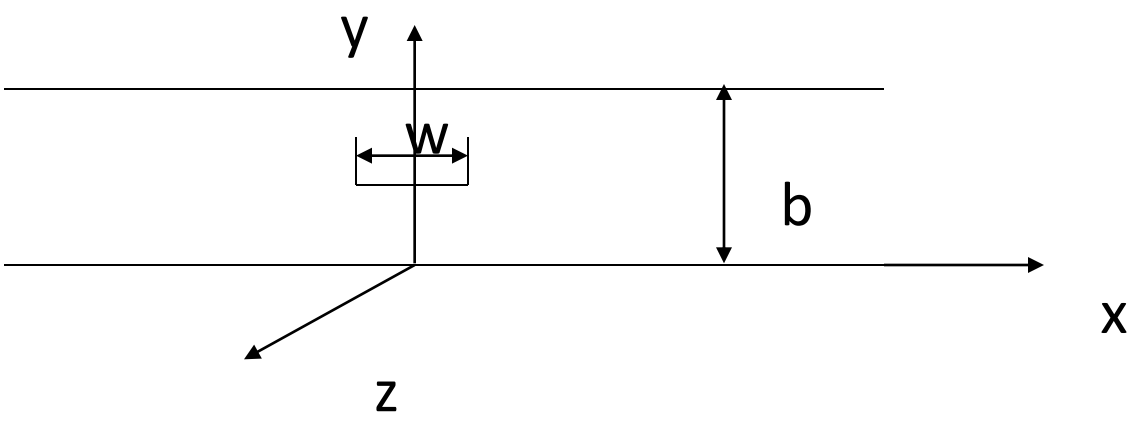
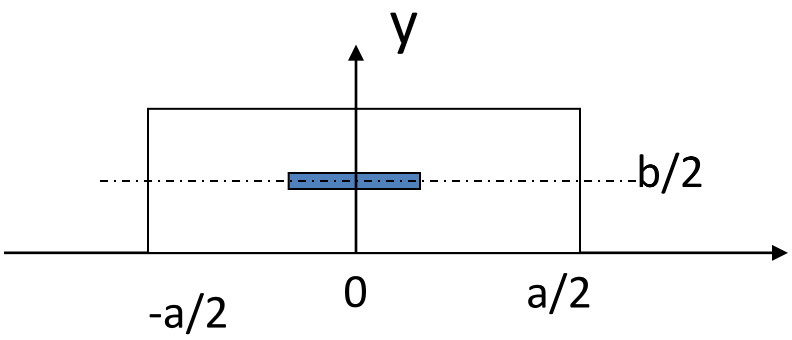
EEC 603
MICROWAVE ENGINEERING

UNIT-1

Stripline



Approximate Electrostatic Solution:



$$\nabla_t^{\,2}\Phi(x,y)=0$$

$$\Phi(x,y)=0 \qquad \text{at } x=\pm a/2$$

$$\& \text{ y} = 0, \text{b}$$

$$\Phi(x,y) = \begin{cases} \sum_{n=1}^{\infty} A_n \cos\frac{n\pi x}{a} \sinh\frac{n\pi y}{a} & \text{for } 0 \le y \le b/2 \\ \sum_{n=1}^{\infty} B_n \cos\frac{n\pi x}{a} \sinh\frac{n\pi (b-y)}{a} & \text{for } b/2 \le y \le b \end{cases}$$

Potential must be continuous at $y = b/2 \Rightarrow A_n = B_n$

$$E_y = -\frac{\partial \Phi}{\partial y}$$

$$E_y = \begin{cases} \sum_{n=1}^{\infty} A_n \left(\frac{n\pi}{a}\right) \cos\frac{n\pi x}{a} \cosh\frac{n\pi y}{a} & \text{for } 0 \le y \le b/2 \\ \sum_{n=1}^{\infty} A_n \left(\frac{n\pi}{a}\right) \cos\frac{n\pi x}{a} \cosh\frac{n\pi (b-y)}{a} & \text{for } b/2 \le y \le b \end{cases}$$

$$\text{Let } \rho_s(x) = \begin{cases} 1 & \text{for } |x| < w/2 \\ 0 & \text{for } |x| > w/2 \end{cases}$$

$$\rho_s(x) = D_y(x, y = b/2^+) - D_y(x, y = b/2^-)$$

$$= 2\varepsilon_0 \varepsilon_r \sum_{\substack{n=1 \\ odd}}^{\infty} A_n \left(\frac{n\pi}{a}\right) \cos\frac{n\pi}{a} \cosh\frac{n\pi b}{2a}$$

$$A_n = \frac{2a \sin(n\pi w/2a)}{(n\pi)^2 \varepsilon_0 \varepsilon_r \cosh(n\pi b/2a)}$$

$$V = -\int_0^{b/2} E_y(x=0, y)\,dy = \sum_{\substack{n=1 \\ odd}}^{\infty} A_n \sinh\frac{n\pi b}{2a}$$

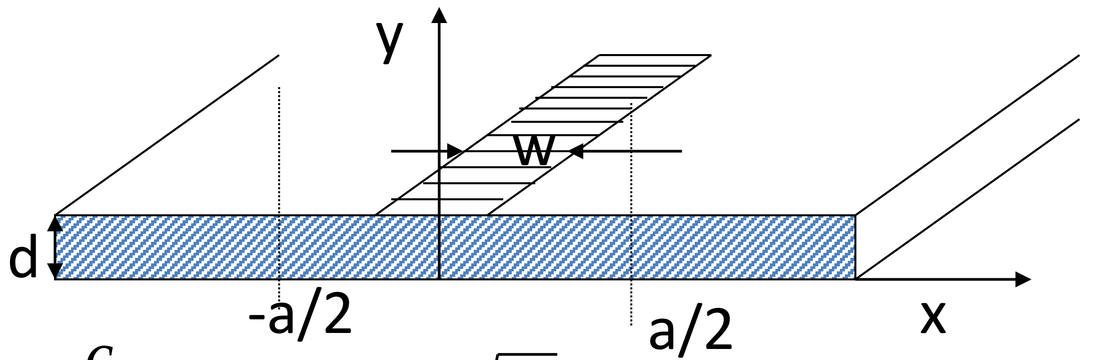
$$Q = \int_{-w/2}^{w/2} \rho_s\,dx = w$$

$$C = \frac{Q}{V} = \frac{w}{\sum_{\substack{n=1 \\ odd}}^{\infty} \frac{2a\sin(n\pi w/2a)\sinh\frac{n\pi b}{2a}}{(n\pi)^2 \varepsilon_0 \varepsilon_r \cosh(n\pi b/2a)}}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{v_p C} = \frac{\sqrt{\varepsilon_r}}{cC}$$

Z_0 is the characteristic impedance

Microstrip



$$v_p = \frac{c}{\sqrt{\varepsilon_e}} \quad , \quad \beta = k_0\sqrt{\varepsilon_e}$$

ε_e is the effective dielectric constant.

$1 < \varepsilon_e < \varepsilon_r$

An Approximate Electrostatic solution

$$\nabla_t^{\,2}\Phi(x,y) = 0$$

$$\Phi(x,y) = 0 \qquad \text{at } x = \pm a/2 \quad ,$$

$$\Phi(x,y) = 0 \qquad \text{at y} = 0,\infty$$

$$\Phi(x,y)=\begin{cases}\sum_{n=1}^{\infty}A_n\cos\frac{n\pi x}{a}\sinh\frac{n\pi y}{a} & \text{for } 0\le y\le d\\ \sum_{n=1}^{\infty}B_n\cos\frac{n\pi x}{a}e^{-\frac{n\pi y}{a}} & \text{for } d\le y\le\infty\end{cases}$$

Potential must be continuous at $y = d \Rightarrow A_n\sin\frac{n\pi x}{a}=B_n e^{-\frac{n\pi d}{a}}$

$$E_y=-\frac{\partial\Phi}{\partial y}$$

$$E_y=\begin{cases}-\sum_{n=1}^{\infty}A_n\left(\frac{n\pi}{a}\right)\cos\frac{n\pi x}{a}\cosh\frac{n\pi y}{a} & \text{for } 0\le y\le d\\ \sum_{n=1}^{\infty}A_n\left(\frac{n\pi}{a}\right)\cos\frac{n\pi x}{a}\sinh\frac{n\pi(d}{a}e^{-n\pi(y-d)/a} & \text{for } d\le y\le\infty\end{cases}$$

$$\text{Let } \rho_s(x) = \begin{cases} 1 & \text{for } |x| < w/2 \\ 0 & \text{for } |x| > w/2 \end{cases}$$

$$\rho_s(x) = D_y(x, y = d^+) - D_y(x, y = d^-)$$

$$= 2\varepsilon_0 \sum_{\substack{n=1 \\ odd}}^{\infty} A_n \left(\frac{n\pi}{a}\right) \cos\frac{n\pi}{a} \left[\sinh\frac{n\pi d}{a} + \varepsilon_r \cosh\frac{n\pi d}{a} \right]$$

$$A_n = \frac{4a \sin(n\pi w / 2a)}{(n\pi)^2 \varepsilon_0 [\sinh(n\pi d / a) + \varepsilon_r \cosh(n\pi d / a)}$$

$$V = -\int_0^d E_y(x = 0, y) dy = \sum_{\substack{n=1 \\ odd}}^{\infty} A_n \sinh\frac{n\pi d}{a}$$

$$Q = \int_{-w/2}^{w/2} \rho_s dx = w$$

$$C = \frac{Q}{V} = \frac{1}{\sum_{\substack{n=1 \\ odd}}^{\infty} \frac{2a\sin(n\pi w/2a)\sinh\dfrac{n\pi d}{a}}{(n\pi)^2 w\varepsilon_{0r}[\sinh(n\pi d/a)+\varepsilon_r \cosh(n\pi d/a)}}$$

$C =$ Capacitance per unit length of the microstrip line with a dielectric constant ε_{r}

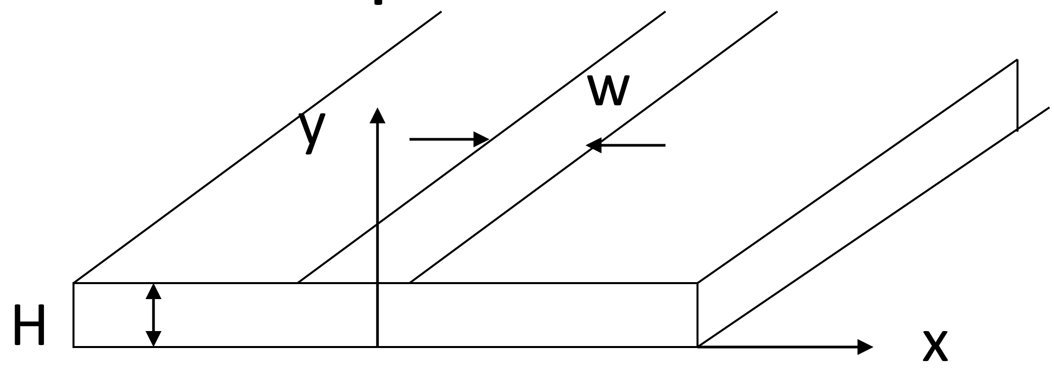
$C_0 =$ Capacitance per unit length of the microstrip line with an air dielectric ($\varepsilon_{\mathrm{r}} = 1$)

$$\varepsilon_e = \frac{C}{C_0}$$

$$Z_0 = \frac{1}{v_p C} = \frac{\sqrt{\varepsilon_e}}{cC}$$

Z_0 is the characteristic impedance

Microstrip Transmission Line



$$\rho(x, y, z) = \rho_s(x, z)\delta(y - H)$$

$$\bar{J}(x, y, z) = \bar{J}_s(x, z)\delta(y - H)$$

$$B = \nabla \times \bar{A}, \qquad \nabla \times \bar{E} = -j\omega\bar{B} = -j\omega\nabla \times \bar{A}$$

$$\bar{E} = -j\omega\bar{A} - \nabla\Phi$$

$$\nabla \times \bar{H} = j\omega\bar{D} + \vec{J}$$

For anisotropic dielectric :

$$\bar{D} = \varepsilon_0\varepsilon_r(E_x\hat{a}_x + E_z\hat{a}_z) + \varepsilon_0\varepsilon_y E_y\hat{a}_y$$

$$\nabla \times \nabla \times \overline{A} = \nabla \nabla \bullet \overline{A} - \nabla^2 \overline{A} = j\omega\mu_0 \overline{D} + \mu_0 \overline{J}$$

$$\overline{D} = -\varepsilon_0 \varepsilon_r [j\omega (A_x \hat{a}_x + A_z \hat{a}_z) + \hat{a}_x \frac{\partial \Phi}{\partial x} + \hat{a}_z \frac{\partial \Phi}{\partial z}]$$

$$- \varepsilon_0 \varepsilon_y (j\omega A_y \hat{a}_y + \hat{a}_y \frac{\partial \Phi}{\partial y})$$

let $\nabla \bullet \overline{A} = -j\omega \varepsilon_0 \varepsilon_r \mu_0 \Phi$ (Lorentz condition)

$$-\nabla^2 \overline{A} = j\omega\mu_0 [-j\omega\varepsilon_0\varepsilon_r \overline{A} + \varepsilon_0 \Phi \nabla \varepsilon_r$$

$$-\varepsilon_0 (\varepsilon_y - \varepsilon_r) \left(j\omega \hat{a}_y A_y + \hat{a}_y \frac{\partial \Phi}{\partial y} \right)] + \mu_0 \overline{J}$$

$\overline{J}$ does not have a y component $\Rightarrow$

$$\nabla^2 A_x + \varepsilon_r(y) k_0^2 A_x = -\mu_0 J_x$$

$$\nabla^2 A_z + \varepsilon_r(y) k_0^2 A_z = -\mu_0 J_z$$

$$\nabla^2 A_y + \varepsilon_y(y) k_0^2 A_y = j\omega\mu_0\varepsilon_0 \left[\Phi \frac{\partial \varepsilon_r}{\partial r} - (\varepsilon_y - \varepsilon_r)\frac{\partial \Phi}{\partial y} \right]$$

$$= j\omega\mu_0\varepsilon_0 \left[(\varepsilon_y - \varepsilon_r)\frac{\partial \Phi}{\partial y} + \Phi(H)\big(\varepsilon_r - 1)\delta(y - H)\big) \right]$$

$$\nabla \bullet \overline{D} = \rho \qquad \Rightarrow$$

$$\varepsilon_{\mathrm{r}}\left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \right) + \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial \Phi}{\partial y} \right) + \varepsilon_r^2 k_0^2 \Phi$$

$$= -\frac{\rho}{\varepsilon_0} + j\omega(\varepsilon_y - 1) A_y(H)\delta(y - H) - j\omega(\varepsilon_y - \varepsilon_r)\frac{\partial A_y}{\partial y}$$

Boundary conditions:

$$\lim_{\tau\to 0}\int_{H-\tau}^{H+\tau}\frac{\partial^2 A_x}{\partial y^2}dy = \left.\frac{\partial A_x}{\partial y}\right|_{H^-}^{H^+} = \lim_{\tau\to 0}\int_{H-\tau}^{H+\tau} -\mu_0 J_{sx}(x,z)\delta(y-H)dy$$

$$= -\mu_0 J_{sx}$$

$$\left.\frac{\partial A_x}{\partial y}\right|_{H^-}^{H^+} = -\mu_0 J_{sx} \quad , \quad \left.\frac{\partial A_z}{\partial y}\right|_{H^-}^{H^+} = -\mu_0 J_{sz}$$

$$\frac{\partial A_x}{\partial y} = -\mu_0 H_z \quad , \quad \frac{\partial A_z}{\partial y} = -\mu_0 H_x$$

$$\left.\frac{\partial A_y}{\partial y}\right|_{H^-}^{H^+} = j\omega\mu_0\varepsilon_0(\varepsilon_r - 1)\Phi(H)$$

$$\left.\frac{\partial \Phi}{\partial y}\right|_{H^+} - \varepsilon_y \left.\frac{\partial \Phi}{\partial y}\right|_{H^-} = -\frac{\rho_s}{\varepsilon_0} + j\omega(\varepsilon_y - 1)A_y(H)$$

In the substrate region away from the interface we have :

$$(\nabla^2 + \varepsilon_r k_0^2) A_x = 0$$

$$(\nabla^2 + \varepsilon_r k_0^2) A_z = 0$$

$$(\nabla^2 + \varepsilon_y k_0^2) A_y = j\omega\mu_0\varepsilon_0(\varepsilon_y - \varepsilon_r)\frac{\partial \Phi}{\partial y}$$

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} + \frac{\varepsilon_y}{\varepsilon_r}\frac{\partial^2 \Phi}{\partial y^2} + \varepsilon_r k_0^2 \Phi = -j\omega(\varepsilon_y - \varepsilon_r)\frac{\partial A_y}{\partial y}$$

In the air region $\varepsilon_r = \varepsilon_y = 1$

For an isotropic substrate $\varepsilon_r = \varepsilon_y$